ABSTRACTS OF ARTICLES DEPOSITED AT VINITI

HEAT EXCHANGE AND HYDRAULIC RESISTANCE IN THE FLOW OF AIR THROUGH A WIRE GRILL

A. A. Basovskaya and V. A. Reisig

UDC 536.24

The efficiency of any heat-exchange apparatus is determined by the maximum attainable heat-flux density, simultaneously allowing for both the efficiency of heat transfer and the dimensions of the surface transmitting or receiving the heat.

A wire grill made of a material with a high coefficient of thermal conductivity, which is in thermal contact with the cooling substance, was analyzed as an efficient slotted heat exchanger. The small weight and size characteristics and the possibility of achieving large heat-flux densities with relatively low hydraulic resistances create real prospects for its use in such fields as radio and quantum electronics and space technology.

In the present report the thermotechnical characteristics of a grill are determined as a function of the geometrical dimensions of its elements. As the subjects we chose copper grills with diameters of $(1, 0.7, 0.5)\cdot 10^{-3}$ m for the wire elements, gaps of $(1, 0.7, 0.5)\cdot 10^{-4}$ m, respectively, and a height equal to $2\cdot 10^{-3}$ m. In this case the blockage factor was constant and equal to $k_b = 0.909$.

The average values for the grill of the coefficient of heat transfer ($\bar{\alpha}$), the Nusselt and Reynolds numbers (Nu and Re), and the coefficient of hydraulic resistance (ζ) were found experimentally. With allowance for the Nusselt numbers obtained, the criterial equation for its value is represented in the following form:

$$\overline{\mathrm{Nu}} = 0.26\,(1 + k_3)\,\mathrm{Re}^{0.6}\mathrm{Pr}^{0.3}.$$

The dependence obtained is valid in the range of $500 \leq \mathrm{Re} \leq 5000$. The Reynolds number was determined for the average velocity of the air stream at the minimum through cross section. The heat flux reached values of $0.15\cdot 10^6 - 0.94\cdot 10^6$ W/m².

Such large heat-flux densities are evidence of the high efficiency of the heat exchanger which we analyzed; the value of q can be increased considerably through a decrease in the diameter and an increase in the height of the grill elements with a proportionate increase in the air flow rate G.

The values of the coefficient of hydraulic resistance in the indicated range of Reynolds numbers were determined. For $\mathrm{Re} \geq 500$ the coefficient ζ = const and only the numerical value differs for all the grills studied.

As a result of the study conducted it was shown that slotted heat exchangers of the type under consideration provide high heat-flux densities with relatively low hydraulic resistance.

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HEAT AND MASS EXCHANGE OF A MAN WORKING AT THE LIMIT OF THERMAL ENDURANCE

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UDC 577.3

The presently existing recommendations on the limits of thermal endurance have not been developed to a sufficient degree, since they do not take into account the nonuniformity of the thermal environment, they are limited by the conditions under which the experiments are performed, and they correspond to the start of the increase in internal body temperature, which does not reflect the limiting thermoregulatory connection between the heat production of the organism and the blood and perspiration supply of the skin surface. It is therefore desirable to obtain such a connection, since it must reflect any limiting variations in the thermal environment.

The aim of the present report is to develop such a connection through the joint use of the proposed theoretical model of heat and mass transfer and of the published experimental data on the allowable variations in the readings of dry-bulb and wet-bulb thermometers in a uniform thermal environment.

We present a system of equations of steady heat and mass exchange in the analyzed model of core body—skin—epidermis—air gap—ambient air medium—radiation source (Fig. 1) and an equation of steady mass exchange. The values of the equivalent coefficients of thermal conductivity are found by integration of the equation of conductive—convective heat transfer in a plane layer containing uniformly distributed heat sources. The coefficient of mass transfer is calculated under the assumptions of complete evaporation of the flow of perspiration from the surface of a film of small thickness and of equality of the thicknesses of the thermal and diffusional boundary layers at the surfaces of the skin and the clothing. The solution of this system of equations results in two equations connecting the temperature t_e of the outer surface of the epidermis, the internal body temperature, the specific thermal resistance r_s of the skin and epidermis, the specific heat production h (normalized to the surface of the skin), the "radiant" temperature of the environment, the coefficients of radiant and convective heat exchange, the heat flux from the radiation sources, the specific vapor content of the ambient air, and the Biot numbers for the layer of clothing and for the air gap between the surfaces of the skin and the clothing. The results of the calculations of $t_e = f(h, r_s)$ are confirmed by the known experimental data for comfort conditions.

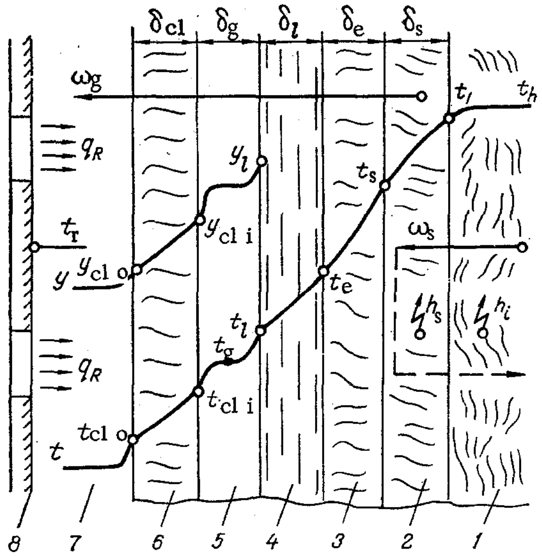

Fig. 1. Calculating scheme for heat and mass exchange: 1) core body; 2) skin; 3) epidermis; 4) perspiration film; 5) air gap; 6) clothing layer; 7) outer air; 8) radiator.

The use of the calculating model in the generalization of data on the limiting characteristics of the ambient air made it possible to obtain a relationship characterizing the limiting indices of thermal regulation:

$$g_p = 2.45 r_s \exp 0.0048 H.$$

Here g_p is the specific weight flux of perspiration to the skin surface, $kg/m^2 \cdot h$; H is the total heat production of a working man, W.

To test the calculating functions obtained under nonsteady conditions we used known experimental data on the heating of a body beyond the limits of thermal endurance. Agreement is noted between experiment and a calculation using the equation of nonsteady heat exchange with the assumption of quasisteadiness of the temperatures.

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Original article submitted July 5, 1976.

MASS TRANSFER IN MULTILAYER INTEGRATED CIRCUIT STRUCTURES

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UDC 621.382.8-76

The process of mass transfer of moisture has been studied in series 400, 431, and 436 integrated circuit chips with protective coatings of SB1S lacquer and KL4B compound. It follows from microscopic analysis of the multilayer structures that the moisture transfer paths consist of channels formed by capillaries and micropores, located along the interphase boundaries of adjoining rigid and elastic materials composing the chip.

Moisture absorption and liberation occur most intensely in the first several tens of hours after the chip is placed in a wet or dry chamber. Moisture exchange of the compound itself can be neglected. The dependence of relative moisture content u on temperature T and time τ spent in the chamber indicates that module moisture absorption at $\tau \geqslant$ 200 h increases nonlinearly with increase in T. Dimensions of the moisture propagation zone within the micromodule at a specified T may be determined from the expression

$$\rho = \varphi \rho_0 \operatorname{erfc} \frac{x}{2\sqrt{a\tau}}, \tag{1}$$

where ρ is the moisture content within the micromodule; ρ_0, density of saturating water vapor; a, coefficient of moisture diffusion; x, distance from surface of contact between different materials. The value of a depends significantly on temperature, changing from $1.2 \cdot 10^{-8}$ to $1.1 \cdot 10^{-10}$ cm^2/sec with increase in T from 20 to 70°C. Results of moisture transfer studies using moisture-sensitive resistors revealed that zones of increased moisture content appeared within the modules studied at the boundaries between different materials. Then as moisture penetrates, these zones expand and reach the central part of the micromodule.

The process of moisture absorption and liberation within the multilayer structures studied is of an exponential character, and may be described by the analytic equations

$$u = u_e \left[1 - \exp\left(-\frac{\mu}{\omega_n} \tau \right) \right], \tag{2}$$

$$u' = u_e \left[\frac{\varphi_1}{\varphi} + \left(1 - \frac{\varphi_1}{\varphi} \right) \exp\left(-\frac{\mu}{\omega_n} \tau \right) \right], \tag{3}$$

where $u_e = \varphi \omega_n / m_0$; $\mu = js$, moisture permeability of the micromodule; j, moisture flux; s, area of micromodule surface; φ, relative humidity of air; ω_n, equilibrium moisture content. The ratio μ/ω_n for the structures studied lies within the range $5 \cdot 10^{-3}$ to 10^{-2} h^{-1} depending on T. The parameters μ and ω_n are determined by the construction of internal structures and depend on the physicochemical properties of the adjacent materials. Thus, protection of the module by only one lacquer layer ~0.1 mm thick reduces the value of μ in comparison to an unprotected specimen by a factor of about two times. A double lacquer protective coat further reduces μ to $(0.5-0.8)\cdot 10^{-3}$ g·h^{-1} and makes the multilayer structure comparable in moisture permeability to an equivalent structure hermetized by a compound layer ∿1.8 mm thick.

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Original article submitted November 30, 1976.

HYGROTHERMAL EQUILIBRIUM OF VEGETABLE FIBERS

V. A. Zagoruiko and Yu. I. Krivosheev UDC 677.1/.2:543.81

Hygrothermal equilibrium of a number of vegetable fibers has been studied experimentally. In the majority of cases the results of such studies have been presented in the form of one or several isotherms [1]. Design and development of drying devices and air conditioning systems require reliable data on equilibrium moisture content u over a wide range of temperature t and relative humidity φ.

Using the thermodynamic sorption equations [2] and the principle of similarity of hygroscopic properties of materials [3] a method is proposed for determining equilibrium moisture content. The method is based on the temperature and interspecies invariance of the characteristic curve

$$w = f(H/H_{0s}), \tag{1}$$

where w is the corrected moisture content, determined by the ratio $w = u/u_{0s}$; H, width of an equivalent pore at the meniscus level of absorbed moisture for specified values of t and φ; u_{0s}, H_{0s}, equilibrium moisture content and width of an equivalent pore at t = 0°C and φ = 100%.

Characteristic curve (1) is constructed on the basis of experimental data

$$u_{\exp} = f(\varphi, t) \tag{2}$$

and the function

$$H/H_{0s} = f(\varphi, t), \tag{3}$$

calculated with the aid of a generalized capillary condensation equation [2].

The studies performed show that the equilibrium content of wood and various vegetable fibers can be calculated with a characteristic curve, Eq. (1), common to all these materials. The mean relative deviation of calculated data from reliable experimental points does not exceed 2.5%, with a maximum deviation of 4-5%.

It should be noted that the temperature invariance of characteristic curve (1), constructured with our calculated equation (3), was also confirmed by experimental data on equilibrium moisture content of several types of grain and dried fruit.

The study considers the method of calculating u_{0s} and generalizing experimental data to one group of hygroscepically similar materials in detail.

Use of the proposed method revealed that it permits use of only a limited number of experimental points (in some cases, practically only one point) to perform calculations with accuracy sufficient for engineering purposes to determine equilibrium moisture content of material over a wide range of moist air parameters.

The study presents tables with data needed for calculation of equilibrium moisture content in various vegetable fibers.

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Original article submitted December 6, 1976.

THERMOPHYSICAL PROPERTIES OF LIGHTWEIGHT ALUMINOSILICATE REFRACTORIES UPON RAREFACTION OF A GASEOUS ATMOSPHERE

E. Ya. Litovskii, Ya. A. Landa, and N. A. Puchkelevich

UDC 666.76.017:536

The thermal conductivity and diffusivity of Soviet and foreign aluminosilicate refractories have been studied in a nitrogen atmosphere in the temperature range 200-1600°C at pressures of 10^{-2} to 760 mm Hg.

The studies revealed that thermal-diffusivity curves at various rarefactions are located approximately equidistantly over the entire temperature interval. At fixed pressure the change in diffusivity with temperature comprises 25-35%. Depending on pressure, diffusivity varies by 10-20% in a slight vacuum and 30-50% at maximum rarefaction. Such a dependence on pressure of the gaseous medium is well described by the model and calculated formula presented in [1].

The character of the temperature dependence and qualitative effect of changes in thermal diffusivity, and hence, conductivity obtained here agree with analogous studies [2, 3] of fireclay and corundum products.

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Original article submitted November 23, 1976.

MASS EXCHANGE BETWEEN MOVING DROPLETS AND A MEDIUM WITH TRANSITIONAL REYNOLDS AND PECLET NUMBERS. EXTERNAL AND INTERNAL PROBLEMS

V. Ya. Rivkind and G. M. Ryskin

UDC 532.72:669.015.23

Variable direction grid systems were used to study mass transfer in uniform motion of spherical droplets ($1 \leqslant$ Re $\leqslant 100$; $1 \leqslant$ Pe $\leqslant 1000$). Preliminary calculations were made [1] of the velocity field inside and outside the droplets, determined by the Reynolds number Re_2 and the ratio of viscosities of the dispersed and continuous phases $\mu = \mu_1/\mu_2$.

Cases were considered in which the resistance to mass transfer is concentrated either in the continuous phase (external problem) or in the dispersed phase (internal problem).

For the external problem calculations revealed that the presence behind the droplets at sufficiently high Re_2 and μ of zones of retrograde flow [1] strongly affected the concentration field; there is a tendency toward development within this zone of a second diffuse boundary layer, with the point of flow incidence serving as the rear critical point (see also [2]).

At sufficiently high Peclet numbers the contribution of the rear zone to the total diffusion flow is extremely small, which allows use for the process as a whole of an established mean Nusselt number $Nu_2 \approx \sqrt{v_{max}Pe_2}$ with establishment time $\tau_r \approx 1/(v_{max}Pe_2)$, which follows from the theory of a diffuse boundary layer and is valid, strictly speaking, only for the forward portion of the droplet surface. Here v_{max} is the maximum velocity on the droplet surface (Fig. 1).

The calculated values of established mean Nusselt number agree well with known experimental data.

Comparison with solution of the external problem in the diffuse boundary-layer approximation [2] shows that the latter gives satisfactory accuracy at $Pe_2 \geqslant 300$.

In the case of the internal problem, at sufficiently high Peclet numbers the diffusion flow distribution over the drop surface shows that in the initial stage of the process there is formed within the drop a diffusion boundary layer similar to that of the external problem (and the same estimates are valid); however thereafter liquid leaving the end section of this layer begins to penetrate into the initial section, leading to a fall in the diffusion flow in the initial section. As a result the diffusion flow distribution approaches symmetry relative to the drop equator; the hypothesis of a boundary layer is no longer necessary, and the Kronig—Brink model becomes valid. Changes in the flow within the drop, depending on Re_2 and μ may be divided into two types: changes in flow line configuration and changes in velocity values. The latter may be described with the aid of a characteristic velocity, for which it is convenient to use v_{max}.

Calculations show (see also [1]) that the change in flow line configuration within the droplet at transitional Reynolds number is insignificant and exerts a second-order influence on the integral characteristics of mass transfer. Introduction of a modified Peclet number

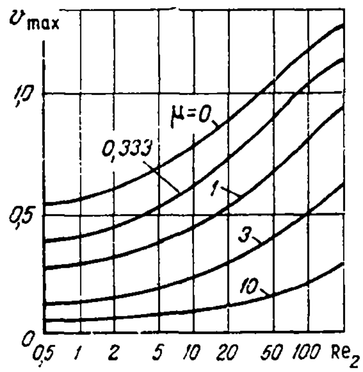

Fig. 1. Maximum velocity value on droplet surface.

defined by $Pe_1^* = Pe_1 v_{max}$ permits consideration of the basic effect of Re_2 and μ and reduction of the problem (with accuracy sufficient in practice) to the analogous problem based on Hill or Adamar – Rybchinskii hydrodynamics and described in detail in the literature.

From this there follows, in particular, the applicability of the Kronig – Brink solution at transitional Reynolds numbers, which agrees with experimental data.

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Original article submitted May 18, 1976.

PROFILE STABILIZATION OF FLOW OF A MEDIUM WITH TEMPERATURE-DEPENDENT HEAT-LIBERATION AND TRANSFER COEFFICIENTS WITHIN THE QUASI-ONE-DIMENSIONAL PLANAR BOUSSINESQ APPROXIMATION

S. M. Babenko, P. P. Lazarev, and A. S. Pleshanov

UDC 536.24:532.54

A theoretical study has been performed of the flow within a planar channel of constant cross section of an ideal Newtonian gas, for which the viscosity coefficient and thermal conductivity are power functions of temperature T and the internal heat liberation $Q \sim 1/T$. It is assumed that the Mach numbers are small and the Reynolds and Peclet numbers are large. The stationary hydrodynamics equations in the quasi-one-dimensional form are separated with respect to the small parameter M^2. Integrals in general form are presented for the stabilized case.

The transitional process produced by inclusion of heat liberation at the channel input, where a stabilized flow at Q = 0 is incident, is studied. Using a model problem it is shown that in the given case a flow core and boundary layers are absent, so that flow readjustment as a whole occurs from section to section. Profiles of velocity u and T are taken from the stabilized solution in implicit form, where the role of the natural transverse coordinate is played by T and the desired functions are values of u and T on the channel axis. The profiles selected satisfy the boundary conditions ($u_w = 0$, T_w = const) as well as the initial conditions, and transform to stabilized profiles at infinity. The non-one-dimensional coefficients which appear upon averaging are functions only of the ratio of temperatures on the axis and channel wall. Both the integrals for the stabilized case and the non-one-dimensional coefficients are expressed in concrete cases by elementary functions together with a probability integral.

Graphs are presented for the stabilized case showing the dependence of the dimensionless values of T, u, thermal flux, and the resistance coefficient on the heat-liberation criterion. Graphs are presented of the distribution of dimensionless T and u over channel length on the channel axis and of dimensionless pressure decrease.

The increase in gas velocity along the channel is a consequence of a decrease in its density due to heating with a constant value of mass flow. Decrease in the pressure gradient along the channel is explained by gradual decrease of this increase in velocity. A decrease was observed in the conditional stabilization line with increase in heating and increase in temperature dependence of the transfer coefficient. This result is not an obvious one, since although the initial slopes of the distributions of all quantities over channel length do increase with increasing heat liberation, the stabilized values of these quantities grow simultaneously. Additional analytic study of the model stabilization measure confirms this computed result. In such a case the value of stabilization length obtained in the model problem with constant transfer coefficients and Q = const is an estimate above the stabilization line.

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Original article submitted September 10, 1976.

PROPAGATION OF A HEAT PULSE IN A LIQUID STREAM

S. E. Kirillov, S. L. Moskovskii, and G. A. Sokolov

UDC 536.22

The existing methods of determining the flow rates of liquids in pipelines using thermal labels are based on the assumption that the velocity of propagation of the pulse is equal to the stream velocity, and they ignore the effect of the heat conduction of the liquid.

To allow for this effect let us consider the equation of propagation of a heat pulse in a liquid moving with an average velocity u through a cylindrical pipe of radius R:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}=a\left(\frac{\partial^2 T}{\partial x^2}+\frac{1}{r}\cdot\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}\right). \tag{1}$$

Here a is the coefficient of thermal diffusivity of the liquid; T is the temperature; t is the time; and x and r are the axial and radial coordinates, respectively.

The solution of Eq. (1), obtained on the assumption of a rectangular shape of the heat pulse of size $(T_0 - T_W)$ acting in a zone of length l and of constancy of the temperature T_W of the wall of the pipe, has the form

$$T=AB+T_W, \tag{2}$$

$$A=2\sum_{n=1}^{\infty}\frac{J_0\left(k_n\frac{r}{R}\right)}{k_n J_1(k_n)}\exp\left[-\left(\frac{k_n}{R}\right)^2 at\right], \tag{3}$$

$$B=\frac{T_0-T_W}{2}\left[\operatorname{erf}\left(\frac{x-ut+l}{2\sqrt{at}}\right)-\operatorname{erf}\left(\frac{x-ut}{2\sqrt{at}}\right)\right], \tag{4}$$

where k_n are the successive roots of the equation $J_0(k) = 0$.

In this solution the factor B represents the solution of the corresponding problem allowing only for axial heat conduction, which can be considered as a first approximation in a calculation of the calibration characteristics of thermal-label flowmeters having point heat collectors. The factor A allows for the cooling of the label owing to heat losses through the wall of the pipe, which is especially important when a distributed heat collector is used.

Calculations of the calibration characteristics and label "lifetime" based on Eqs. (2)-(4) showed good agreement with experimental data obtained on a specially developed installation.

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Original article submitted August 24, 1976.

NONSTEADY OPERATION OF AN INJECTOR DISCHARGING GAS INTO A SYSTEM HAVING A DISTRIBUTED LEAK

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UDC 621.694.2

The operation of the injector is explained schematically by Fig. 1. The high-pressure injecting gas is supplied through the fast-acting valve 1 (after it is opened) to the nozzle 2. Passing through the nozzle, it injects the low-pressure gas, mixing with it in the cylindrical mixing chamber 3. After the diffuser 4 the stagnant mixed gas enters the vessel 5 with porous walls, which encloses a medium with excess pressure.

Using the usual model of the mixing of gas jets in a cylindrical mixing chamber, but with allowance for the unsteadiness, we write the laws of conservation of mass and momentum in integral form for the mixing chambers as follows:

$$l\frac{d\bar{\rho}}{dt} + \rho_3 u_3 - \rho_2 u_2 [1 - (d_1/d_3)^2] - \rho_1^* u_1^* (d_1^*/d_3^*)^2 = 0, \tag{1}$$

$$l\frac{d(\overline{\rho u})}{dt} + \rho_3 u_3^2 - \rho_1 u_1^2 (d_1/d_3)^2 - \rho_2 u_2^2 [0.5 - (d_1/d_3)^2] = p_{02} - p_3. \tag{2}$$

The motion of the gas in the diffuser is described using the laws of conservation of mass and energy

$$\frac{L}{12}[1 + d_3/d_4 + (d_3/d_4)^2]\frac{d\bar{\rho}}{dt} + \rho_4 u_4 - \rho_3 u_3 (d_3/d_4)^2 = 0, \tag{3}$$

$$\frac{L}{12}[1 + d_3/d_4 + (d_3/d_4)^2]\frac{d(\overline{\rho u^2})}{dt} + \rho_4 u_4^3 - \rho_3 u_3^3 (d_3/d_4)^2 = 2[p_3 u_3 (d_3/d_4)^2 - p_4 u_4]. \tag{4}$$

For the vessel with porous walls we can write

$$V\frac{d\rho}{dt} = \frac{\pi}{4}\rho_3 u_3 d_3^2 - \rho_5 vF, \tag{5}$$

where, in accordance with the Darcy law,

$$v = \xi (p_5 - p_e). \tag{6}$$

In Eqs. (1)-(5) the quantities with an upper bar are averaged over the volumes for which these equations are written. The quantity v and quantities u, ρ, and p with indices other than 1 and e are functions of time.

The system of equations (1)-(6) with the appropriate initial conditions was solved for the case of a supersonic low-pressure injector in which homogeneous gases with the same stagnation temperatures are mixed. The solution for the dimensionless pressure drop at the porous wall has the form

$$(p_5 - p_e)/p_e = a_3/a_2 + A \exp(-a_1 t/2) \cos(\lambda t/2 + \varphi)$$

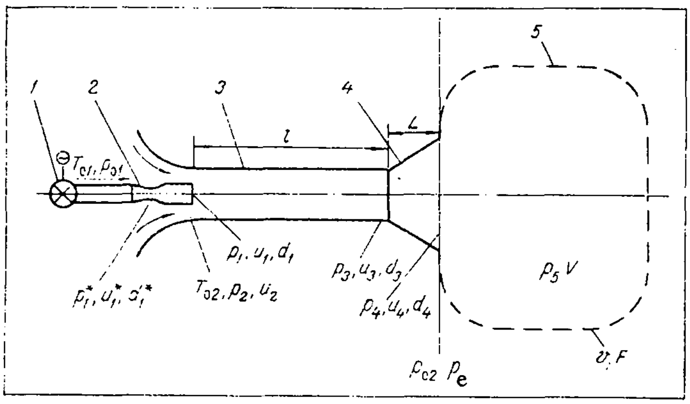

Fig. 1. Diagram of the discharge of gas by an injector into a vessel with porous walls.

and corresponds to damped driven oscillations in the presence of a driving force which is constant in magnitude. The coefficients a_1, a_2, a_3, λ, and φ depend on the operating and structural parameters of the injector and the filtration system, which figured in the initial equations.

NOTATION

p, pressure; u, axial component of gas velocity; v, filtration velocity; d, diameter; l, length of cylindrical mixing chamber; L, length of diffuser; F, area of porous surface; V, volume of vessel with porous walls; ρ, gas density; t, time; ξ, filtration coefficient. Indices: 1, injecting gas; *, critical cross section; 2, injected gas; 3, exit cross section of mixing chamber; 4, exit cross section of diffuser; 5, vessel with porous walls; e, medium external to vessel; 0, stagnation of stream.

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Original article submitted September 6, 1976.

HYDRODYNAMIC STABILITY OF PLANE GRADIENT FLOW OF A NON-NEWTONIAN FLUID WITH A HIGHLY VISCOUS CORE

A. S. Romanov

UDC 532.51:532.135

Hydrodynamic stability is analyzed for the case of plane gradient flow of a non-Newtonian, structurally viscous medium exhibiting nonlinear mechanical properties at stresses below the shear strength τ_0. The dimensionless rheological law relating the deviator of the dimensionless stress tensor σ_{ij} to the dimensionless strain-rate tensor f_{ij} can be written, for example, in the form

$$\sigma_{ij} = 2\eta(\omega) f_{ij},$$

where $\eta(\omega)$ is the variable apparent viscosity of the medium and $\omega \equiv \sqrt{2f_{ij}f_{ij}}$. If the intensity of the strain-rate tensor is greater than a certain value $\omega > \omega^*$, then $\eta(\omega)$ coincides with the analogous dependence for a Shvedov—Bingham viscoplastic fluid: $\eta(\omega) = 1 + \varkappa/\omega$ ($\varkappa$ is the plasticity parameter). Then the case is discussed in which the characteristic strain rate is much greater than the value at which destruction of the supramolecular structure takes place; i.e., $\omega^* \ll 1$. Here stationary flow scarcely differs from its counterpart for a viscoplastic fluid. Two flow zones are formed in the channel: 1) a highly viscous central zone, in which the strains are negligible ($\omega < \omega^* \ll 1$); 2) a viscous-flow zone, in which $\omega \sim 1$.

The evolution of velocity perturbations periodic in time and along the length of the channel is an eigenvalue problem and is reduced in the linear approximation to an appropriate secular equation. The latter depends significantly on the width of the highly viscous flow core. If $\varkappa \to 0$, the problem passes to the limit of Newtonian viscous flow. Numerical computations of the critical flow parameters as a function of the plastic properties of the medium are carried out on the basis of the relations derived in the paper. It is found that stability is virtually unaffected by the singularities of the mechanical behavior of the investigated media at small shear rates. A comparison of the data obtained in the study with experimental data on the flow stability of high-paraffin petroleum and other viscoplastic media in circular pipes indicates good qualitative agreement.

Dep. 1632-77, April 4, 1977.
Original article submitted November 2, 1976.

TEMPERATURE FIELD IN A REINFORCED-CONCRETE WALL FORMED IN A METAL MOLD

V. A. Makagonov UDC 536.2

The growing number of buildings being constructed from monolithic concrete has created a need in structural heat physics for analysis of the temperature fields in structures formed in metal molds.

Analytical solutions of a number of practical problems are given in the paper for the example of a flat structure sheathed in a metal casing.

Cauchy boundary conditions are specified on the surfaces. The ambient temperature is assumed to be constant, corresponding to the monthly average ambient air temperature, or to vary harmonically (general case).

The thermal conductivity of the metal layers is much greater than that of the concrete, and their thickness is much less, so that the temperature field in the metal can be neglected. Ideal thermal contact is assumed between the layers. The problems are treated in the one-dimensional setting. Analytical expressions are also given for the temperature field in thin-walled structures such that the exothermy of concrete can be neglected.

The solutions are obtained by means of the Laplace integral transform.

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Original article submitted May 4, 1976.

TEMPERATURE FIELD OF A THREE-LAYERED ASYMMETRIC PLATE WITH BOUNDARY CONDITIONS OF THE THIRD KIND

A. I. Ventskunas, I. I. Navasaitis,
and P. A. Ramelite UDC 536.21

In thermal calculations one often encounters problems on the determination of the temperature fields in multilayered structures in which several materials with different thermophysical characteristics are in thermal contact. The temperature of the surrounding medium is a variable quantity.

The one-dimensional nonsteady temperature field of a three-layered asymmetric plate whose width and length are infinitely large in comparison with the thickness and which exchanges heat with the surrounding medium in accordance with the Newton – Richmann law is analyzed in the report. The differential equation of heat conduction is solved using operator calculus. An expression is obtained for the temperature fields for three cases: when the temperature of the surrounding medium is constant, when the temperature of the medium varies by a linear law, and when the temperature of the medium varies by an exponential law. The expressions obtained are suitable for calculation on a computer.

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COOLING OF AN INFINITE CYLINDRICAL HEAT SOURCE IN AN ISOTROPIC MEDIUM

I. M. Kutasov

UDC 536.242

An approximate equation is obtained describing the relative temperature distribution in a homogeneous medium around a cylindrical heat source with a constant temperature:

$$v(R, \mathrm{Fo}) = e^{-A(R-1)-B(R-1)^2}; \tag{1}$$

the values of A(Fo) and B(Fo) are presented in the report. The temperature along the axis of the source after it is turned off is found through a fundamental solution describing the cooling of a cylindrical body with a known initial temperature distribution located in a medium with a constant temperature

$$V(\mathrm{Fo}, n) = 1 - e^{-p} + 2pe^{-p}\{2\beta + (1-2\beta\gamma)\sqrt{\pi\beta}\, e^{\gamma^2\beta}[1-\Phi(\gamma\sqrt{\beta})]\}, \tag{2}$$

where

$$p = \frac{1}{4n\mathrm{Fo}}, \quad \gamma = A + 2p, \quad \beta = \frac{1}{4(B+p)}, \quad \mathrm{Fo} = \frac{at_1}{r_0^2}; \quad n = t_2 : t_1.$$

NOTATION

r_0, radius of heat source; t_1, duration of action of source; t_2, time measured after turning off of source; $\Phi(x)$, error probability integral. The thermophysical parameters of the heat source and the medium are assumed to be the same.

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Original article submitted March 25, 1975.

ONE METHOD OF ANALYSIS OF THERMAL PROCESSES IN BODIES OF SIMPLEST SHAPE

A. I. Fesenko

UDC 536.2

A method of calculating the statistical characteristics of the temperature field in bodies of the simplest shape is analyzed in the report; the method is based on the heat-transfer function and the principles of a complex exponent and a displacement pulse with allowance for the random character of the technical realization of the boundary function $U(\tau)$ in the course of the experiment and for the additive error

$$\varepsilon(\tau) = \tilde{U}(\tau) - U(\tau)$$

with a correlation function $K(\tau, \tau')$. Here $\tilde{U}(\tau)$ is the boundary function realized by technical means in conducting thermophysical tests.

The mathematical expectation of the temperature field is analyzed as the sum of the free, induced, and resonance motions of the temperature field in the body.

It is shown that for the determination of the free motion of the temperature field it is desirable to analyze the Laplace transform of the boundary function $U(\tau)$ as the heat-transmission function of a fictitious body. The induced and resonance motions of the temperature field are calculated by a unified method with allowance for the theorem of an equivalent signal.

The dispersion and the correlation function of the temperature field for a one-dimensional body are calculated with allowance for [1, 2] by the equations

$$D(r, \tau) = \int_0^\tau \int_0^\tau g(r, \tau - t)\, g(r, \tau - t')\, K(t, t')\, dt dt',$$

$$K_T(r,\tau,\tau')=\int_0^{\tau}\int_0^{\tau} g(r,\tau-t)\,g(r,\tau'-t')\,K(t,t')\,dt\,dt',$$

where $g(r, \tau)$ is the inverse transform of the heat-transfer function and r is the spatial coordinate.

The spectral density of the temperature field is determined with allowance for [1]. The results obtained can be used to determine the statistical characteristics of the temperature field in the studied materials in experimental thermophysics when electrical stabilizing or programmed regulators are used.

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Original article submitted September 9, 1976.

OPTIMIZATION OF DIMENSIONS OF HEAT CONDUCTORS

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UDC 621.396.6-181.48.017.7

The influence of the geometry and Biot number on the thermal resistance of heat conductors in the form of a rectangular bar and a cylindrical disk is investigated theoretically.

Solutions are obtained for the Laplace temperature equation $\Delta T = 0$ by separation of variables under the following boundary conditions:

for the bar

$$\left.\frac{\partial T(x,y)}{\partial x}\right|_{x=\pm L}=0,\quad \left.\frac{\partial T(x,y)}{\partial y}\right|_{y=0}=\begin{cases}-q/\lambda, & \text{if}\quad |x|\leqslant l,\\ 0, & \text{if}\quad |x|>l,\end{cases}$$

$$\alpha(T-T_\infty)=-\lambda\partial T/\partial y\quad\text{for}\quad y=\delta;$$

for the disk

$$\frac{\partial T}{\partial z}=\begin{cases}0, & \text{if}\quad \rho>r,\\ -q/\lambda, & \text{if}\quad 0\leqslant\rho\leqslant r,\end{cases}$$

$$\frac{\partial T}{\partial \rho}=0\ \text{for}\ \rho=R\text{, and}\ 0\leqslant z\leqslant\delta,$$

$$\lambda\frac{\partial T}{\partial z}=-\alpha(T-T_\infty)\ \text{for}\ 0\leqslant\rho\leqslant R,\ z=\delta.$$

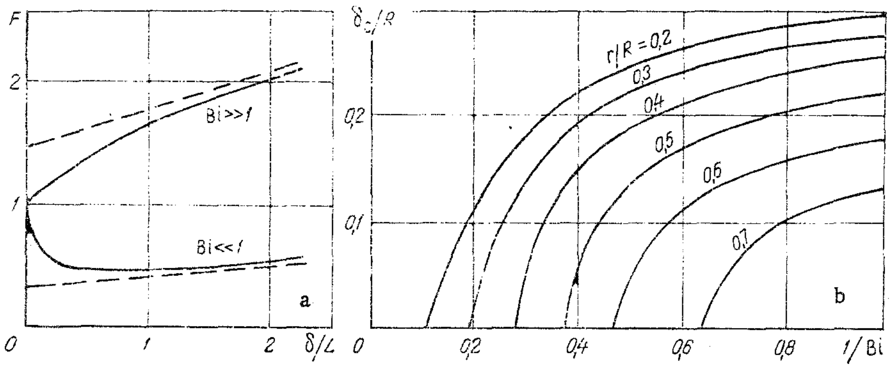

Fig. 1. Dimensionless thermal resistance and optimum thickness of heat conductors.

An analysis of the relations obtained for the dimensionless thermal resistance of heat conductors, expressed as $F = \alpha[T(\rho = 0, z = 0) - T_\infty]/q$ for the disk and $F = \alpha[T(x = 0, y = 0) - T_\infty]/q$ for the bar, indicates the existence of a minimum F when the Biot number ($Bi = \alpha R/\lambda$ for the disk) is sufficiently small. The variation of F as a function of the relative thickness of the bar (δ/L) and the Biot number is shown in Fig. 1a. For $\delta \gg L$ the thermal resistance depends linearly on the bar thickness; i.e., the curves $F(\delta)$ have asymptotes (indicated by dashed lines in Fig. 1a). For small Biot numbers the minimum value of F must be observed for a finite thickness (δ_0) of the bar. The behavior of the curves $F(\delta)$ is similar for the disk, where now δ/L corresponds to δ/R.

Relations are given for determining the optimum thickness of a bar and a disk (δ_0) for minimization of their thermal resistance (for fixed values of l/L and Bi or of r/R and Bi). The thermal resistances of heat conductors are calculated over a wide range of parameters on a Minsk-32 digital computer. The results of the calculations of δ_0 for a disk with $r/R = 0.2$ to 0.7 are given in Fig. 1b. For a given value of the parameter r/R there is a certain threshold number Bi (Bi_c) such that for $Bi \geq Bi_c$ the minimum F is observed only for $\delta_0 = 0$. The value of Bi_c decreases monotonically with increasing r/R.

Approximate (within 10 to 20% error limits) relations are given for the determination of δ_0.

NOTATION

α, heat-transfer coefficient, $W/m^2 \cdot {}^\circ K$; q, heat flux density, W/m^2; T, temperature of heat conductor, °K; T_∞, ambient temperature, °K; $2l$, width of bar heating strip, m; r, radius of disk heating zone, m; L, width of bar, m; R, radius of disk, m; δ, thickness of heat conductor, m; λ, thermal conductivity of conductor material, $W/m \cdot {}^\circ K$.

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Original article submitted November 30, 1976.

MATHEMATICAL MODELING OF DIFFUSION PROCESSES IN APPARATUS WITH COUNTERCURRENT–CROSSCURRENT MOTION OF SOLID AND LIQUID PHASES

L. N. Pavlov

UDC 669.712.1:532.73

The author derives an equation that is formally identical to the equation of Todes and Bikson [1], but, unlike the latter, it can be used for a linear distribution ratio and does not suffer from the contradictions associated with the apparent validity of the relation of Ya. B. Zel'dovich [1] in the asymptotic stage when the parabolic quasilinear system of differential equations is replaced by a hyperbolic system.

A system of equations in the form (1) or in the dimensionless form (2) is proposed on the basis of a theoretical analysis of the diffusion process in a belt-type moving-bed percolation apparatus, permitting engineering calculations to be carried out for the latter:

$$c_{0i} = \frac{a_0}{\Gamma}(1 - b_{i+1} + b_i) + \sum_{m=1}^{i} c_{0m}(b_{i+2-m} - b_{i-m}) + c_{0i+1}(b_1 - b_0), \tag{1}$$

where

$$b_m = \frac{1}{\tau_0}\int_0^{m\tau_0} y_L(\tau; H)\, d\tau \quad \text{and} \quad b_0 \equiv 0,$$

$$y_L(\tau; \xi) = \exp(\tau - \xi)\left(\sum_{n=0}^{\infty}\frac{\xi^n}{n!}\sum_{k=n}^{\infty}\frac{\tau^k}{k!}\right),$$

$H = \beta_e h/v$, $\tau = \beta_e t'/\Gamma$, $t' = t - x/v$, $\xi = \beta_e x/v$, and h is the height of the layer in the container. The function $y_L(\tau; \xi)$ is the solution of the mass-transfer dynamical problem.

We write the system (1) in the dimensionless form

$$\sum_{m=1}^{i} u_m [b_{i+2-m} - 2b_{i+1-m} - b_{i-m}] - u_i + u_{i+1} b_1 = -(1 - b_{i+1} + b_i), \tag{2}$$

where $u_m = c_{0m}\Gamma/\alpha_0$ is the dimensionless concentration.

It is shown that if the process in the two-phase element of the apparatus is described by a linear system of partial differential equations, the computational algorithm remains unchanged, regardless of the type of system (hyperbolic or parabolic) and the governing process rate of the stage. The foregoing considerations affect only the technique used to determine the coefficients of the system of algebraic equations.

A rapid and reasonably accurate method of numerical inversion of the functional transform is developed for engineering calculations. Several mathematical models are analyzed, and the necessary transforms of functions are obtained for calculations of the process in numerous specific situations. The proposed mathematical models can be used either for engineering calculations and optimization of the process, as well as for construction of the apparatus in the design stage, or for the synthesis of automatic control systems.

NOTATION

c_i, α_i, concentrations of retrievable component in moving and stationary phases, respectively; v, and w, velocities of moving and stationary phases; Γ, distribution ratio (Henry constant); β_e, mass-transfer coefficient; t, total leaching-out time; x, coordinate of a point in the layer in the direction of motion of the solution; i = 1, 2, 3, ..., n, number of distributing zones of solution; j = 1, 2, 3, ..., m, number of collecting trays (in general, $n \neq m$).

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Original article submitted January 10, 1977.

INFLUENCE OF AN INSOLUBLE SURFACE-ACTIVE FILM ON DYNAMICS OF RADIAL OSCILLATIONS OF A GAS BUBBLE

S. P. Levitskii

UDC 541.24:532.5

The sound-induced pulsations of an isolated bubble with a certain quantity of insoluble surface-active material (SAM) distributed uniformly over its surface are investigated in connection with the modeling of the influence of SAM additives on the dynamical behavior of gaseous inclusions in a liquid. An equation for radial oscillations of the bubble is derived, taking account of the variation of the coefficient of surface tension at the bubble–liquid interface during the oscillatory process:

$$\delta(R\ddot{R} + 3/2\dot{R}^2) + 4\mu\dot{R}/R = p_a \sin\omega t - p_\infty +$$
$$+ [p_\infty + 2\sigma_0 R_0^{-1}](R_0/R)^{3\gamma} - 2\sigma_0 R^{-1}[1 - \nu(R_0^2/R^2 - 1)]. \tag{1}$$

A linear analysis of Eq. (1) shows that with the SAM present on the bubble surface the natural resonance frequency of the oscillations is higher, the oscillation amplitude at resonance is lower, and the phase difference between the bubble and sound pressure oscillations is smaller. As a result, the insoluble surface-active film exerts a damping effect on the radial oscillations of the bubble in the frequency interval $\omega \leqslant \omega_r^0$. For $\omega > \omega_r^0$ a certain amplification of the oscillations is possible due to the resonance shift associated with the increase of ω_r. The results of a numerical analysis of the nonlinear equation (1) corroborate the main conclusions of the linear approximation.

It is shown by an approach similar to that used in [1, 2] that a film of insoluble SAM on the surface of a bubble undergoing small oscillations is equivalent to a thin elastic film. A relationship is established between the constants characterizing the properties of the SAM film and of a thin elastic film. It is shown that the dimensionless surface activity ν of the SAM has the significance of an elastic constant.

NOTATION

ρ, density; μ, viscosity; R, bubble radius; p_a, sound pressure amplitude; ω, frequency; p_∞, static pressure; σ, coefficient of surface tension; γ, polytropy exponent; ω_r^0, linear resonance frequency of bubble without SAM; ν, dimensionless surface activity of SAM. Indices: subscript 0, equilibrium value.

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Original article submitted December 3, 1976.

AN APPROACH TO INTERCHANGEABILITY OF THERMISTORS IN TEMPERATURE MEASUREMENT CIRCUITS

Yu. A. Lozinskii

UDC 621.317.733.024.001.24

The application of bridge circuits incorporating thermistors for the measurement of a wide range of temperatures (0 to 100°C) is discussed. The parameters of a bridge circuit with a linear scale are determined by the point-interpolation method for three scale points corresponding to temperature values T_1, T_0, T_2 [1, 2], and the departure of the scale from linearity is determined according to the equation

$$\delta = \frac{B^2 t_0^3}{18\sqrt{3}\, T_0^4}, \tag{1}$$

in which $t_0 = (T_2 - T_1)/2$, $T_0 = (T_1 + T_2)/2$.

For temperature measurements in a narrow interval, provision is made for interchangeability of the thermistors by the inclusion of corrective resistances [3]. For wide-range temperature measurements, an acceptable interchangeability of thermistors cannot be ensured by means of two or three corrective resistances [4]. Thus, for the calculation of a bridge circuit using MMT-4 thermistors with R_{20} = 3 kΩ and B = 2000 to 400°K the error after correction in the range from 0 to 100°C is greater than 5°C, and the error due to departure of the scale from linearity is 1.8°C.

Consequently, one way to provide thermistor interchangeability is to furnish a set of primary transducers, i.e., rather than switch the sensor to selected corrective resistances, to switch the entire primary transducer circuit in this manner for multiple-point temperature measurements over wide ranges.

The use of bridge-type measurement circuits requires a set of five constant resistances for connection to the sensor; two of them (arms of the bridge) are selected without additional discrimination from the rated value, and the other three (the diagonal resistance and one arm of the bridge, plus a resistance connected in series with the bridge circuit) are selected by an analytical or experimental testing procedure. Thus, in the thermistor bridge circuit it is necessary to select only three resistances with special accuracy. For temperature measurements in the range from 0 to 100°C by means of MMT-4 thermistors (R_{20} = 3 kΩ) the error of the given circuit-interchangeability scheme is 2.1°C. With allowance for the degree of nonlinearity of the scale, the interchangeability error for a given set of thermistors does not exceed 0.3°C.

NOTATION

T_1, T_2, lower and upper limits of measured temperature range; T_0, midpoint of measured temperature range; δ, deviation of indicator scale from linearity; B, thermistor constant, determined by the properties and geometry of the semiconductor.

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Original article submitted September 6, 1976.

CALCULATION OF POTENTIAL FIELDS WITH MIXED BOUNDARY CONDITIONS OF THE THIRD KIND

A. M. Vishnevskii and G. É. Klenov

UDC 536.24

The boundary problem is analyzed for a Laplace equation with mixed boundary conditions of the third kind

$$U(p) - k(p)\frac{\partial U(p)}{\partial n} = f(p),\ p \in S, \tag{1}$$

where S is the boundary surface, n is the inner normal, k and f are known functions of the coordinates of the boundary surface, with $S = S_1 + S_2$ and $k(p) = k_2 = \text{const} \neq 0$ for $p \in S_2$.

Using a representation of the potential in the form

$$U(q) = \frac{1}{k_2}\int_S \psi(p)\, G_3(q, p)\, dS,\ q \in S, \tag{2}$$

where G_3 is a Green function of the third kind for a boundary problem with the boundary conditions

$$U(p) - k_2\frac{\partial U(p)}{\partial n} = \psi(p),\ p \in S, \tag{3}$$

and an identical transformation of (1) in the case of $k(p) \neq 0$ such that

$$\psi(p) = \begin{cases} \dfrac{k_2}{k(p)} f(p) + \dfrac{k(p) - k_2}{k(p)} U(p), & p \in S_1; \\ f(p), & p \in S_2, \end{cases} \tag{4}$$

the initial problem is reduced to the solution of an integral equation of the second kind with a kernel having a weak singularity:

$$U(q) - \frac{1}{k_2}\int_{S_1} U(p)\ \frac{k(p) - k_2}{k(p)} G_3(q, p)\, dS = \int_S \frac{f(p)}{k(p)} G_3(q, p)\, dS,\ q \in S_1. \tag{5}$$

The integration of the unknown function in (5) is carried out over only part of the boundary surface S_1, which considerably simplifies the performance of the numerical calculations (especially in the case of an infinitely extended boundary). After the solution the potential at the boundary at any point $q \in S_2$ can be found from (5) while its normal derivative dU/dn can be found directly from the boundary conditions (1). In this case the potential within the region is determined with the help of the expression

$$U(q) = \frac{1}{2k_2} \int_S \psi(p) \left[G_3(q, p) + k_2 \frac{\partial G_3(q, p)}{\partial n} \right] dS. \tag{6}$$

In the case of $k^{-1}(p) \neq 0$ with $p \in S_1$ the integral equation for the normal derivative

$$k(q) \frac{\partial U(q)}{\partial n} - \frac{1}{k_2} \int_{S_1} \frac{\partial U(p)}{\partial n} [k(p) - k_2] G_3(q, p) \, dS = -f(p) + \frac{1}{k_2} \int_S f(p) G_3(q, p) \, dS \tag{7}$$

is found in a similar way using (1)-(4). After (7) is solved the potential at any point $q \in S$ is determined by the equation

$$U(q) = \frac{1}{k_2} \int_{S_1} \frac{\partial U(p)}{\partial n} [k(p) - k_2] G_3(q, p) \, dS + \frac{1}{k_2} \int_S f(p) G_3(q, p) \, dS.$$

In this case the normal derivative dU/dn at a section of the boundary S_2 is determined from the boundary conditions (1) while the potential at any point within the region is determined from Eq. (6).

As an example illustrating the effectiveness of the proposed method, the temperature field [U(p) = T(p)] is calculated in the half-space Z > 0 with the boundary conditions

$$T(R, Z) - k_1 \frac{\partial T(R, Z)}{\partial Z} = 1, \; Z = 0, \; R \leqslant 1;$$

$$T(R, Z) - k_2 \frac{\partial T(R, Z)}{\partial Z} = 0, \; Z = 0, \; R > 1.$$

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Original article submitted July 27, 1976.

JOULE HEAT RELEASE IN A CYLINDRICAL CHANNEL CONNECTING TWO HALF-SPACES

P. A. Pavlov and Yu. K. Malikov

UDC 537.321

In a half-space — cylinder — half-space region a harmonic-function representation is found which equals +1 at infinity in one half-space and −1 in the other when the surfaces are insulated. For this the potential at the end of the cylinder is expanded in a uniformly converging series by Bessel functions

$$\Psi(R) = B_0 + \sum_1^N B_n J_0(R\beta_n), \; N \to \infty, \; 0 \leqslant R \leqslant 1. \tag{1}$$

The normal derivative of the potential at the end is determined from the known solution for the cylindrical region. This derivative is used to solve the boundary problem of the second kind for the half-space by the Green-function method, with the normal derivative being equal to zero at the plane outside the end of the cylinder by the condition of the problem. The solutions for the cylinder and for the half-space are joined with respect to the potential at the disk $0 \leqslant R \leqslant 1$. As a result, an equation is obtained for the determination of the coefficients B_n:

$$B_0 \left[1 + \frac{2}{\pi l} E(R) \right] + \sum_1^N B_n \left[J_0(R\beta_n) + \frac{2}{\pi} \beta_n \operatorname{cth}(l\beta_n) I_n(R) \right] = 1,$$

$$I_n(R) = \int_0^1 J_0(\beta_n x) K\left(\frac{2\sqrt{Rx}}{R+x} \right) \frac{x\,dx}{R+x}. \tag{2}$$

For a numerical solution of this equation on a BÉSM-6 computer the reduction method is used for N = 47 with 20 values of l. A table is presented for B_n with l = 0.1, 0.25, and 1.0. The error is estimated from the stability of the solution relative to the choice of the joining points.

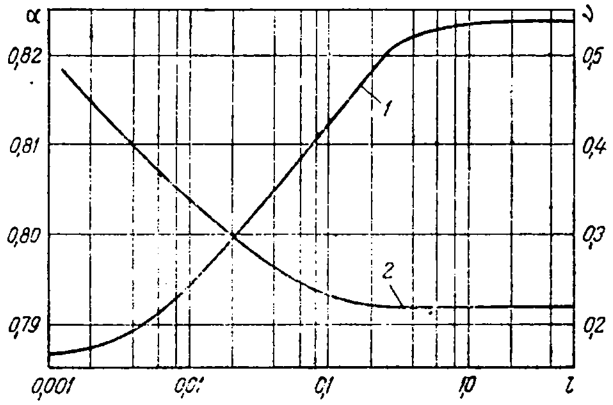

Fig. 1. Dependences of α and ν on l: 1) correction α to length of cylinder; 2) coefficient ν for determination of density of heat release near rim.

An effective cylinder length $2(l + \alpha)$ is obtained such that the total electrical resistance of the region is equal to the resistance of a cylinder of such a length. The distribution of the local power of the release of Joule heat is analyzed in units of power calculated through an average of the electric current density over the cross section of the cylinder. Near the rim at the outlet of the cylinder into the half-space this relative power grows monotonically with approach to the rim by the law

$$q(r) = \left(\frac{2\pi}{3}\right)^2 \nu^2 r^{-2/3}. \tag{3}$$

A recommendation for the allowance for rounding of the rims is given in the article. The coefficients α for $l \geqslant 1$ and $l \ll 1$ and ν for $l \ll 1$ which were found agree with analytical results (Fig. 1).

NOTATION

R, $2l$, dimensionless radius of cylinder and height normalized to the radius; β_n, roots of $J(\beta_n) = 0$; K(x), E(x), complete elliptic integrals of the first and second kinds; r, distance from rim normalized to radius of cylinder.

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A SPECIAL BILINEAR EXPANSION OF THE GREEN FUNCTION FOR PROBLEMS OF HEAT CONDUCTION

I. M. Prikhod'ko

UDC 536.21

In the general case a one-dimensional temperature field is described by the following system of differential equations:

$$\frac{1}{x^\nu}\cdot\frac{\partial}{\partial x}\left[x^\nu \gamma(x)\frac{\partial t(x,\tau)}{\partial x}\right] = \rho(x)\,c(x)\,\frac{\partial t(x,\tau)}{\partial \tau} - \psi(x,\tau); \tag{1}$$

$$a_1 \frac{\partial t(x,\tau)}{\partial x}\Big|_{x=0} + a_2(\tau)\,[t(0,\tau) - t_{c_1}(\tau)] + a_3 q_1(\tau) = 0;$$

$$b_1 \frac{\partial t(x,\tau)}{\partial x}\Big|_{x=\delta} + b_2(\tau)\,[t(\delta,\tau) - t_{c_2}(\tau)] + b_3 q_2(\tau) = 0; \tag{2}$$

$$t(x,0) = f(x). \tag{3}$$

The system (1)-(2) is equivalent to an integral equation of the Fredholm type of the second kind [2]

$$z(x,\tau) = -\int_0^\delta G(x,\eta,\tau)\,\eta^\nu F(\eta,\tau)\,d\eta, \tag{4}$$

where $G(x, \eta, \tau)$ is the Green function, defined as the temperature at the point x due to the action of a steady heat source of unit power located at the point η of a body at the surface of which nonsteady heat exchange occurs with a medium whose temperature is equal to zero.

Through a series expansion of Eq. (4) in a special uniformly converging bilinear series [1] Eq. (4) is reduced to the system of ordinary differential equations [3]

$$A_{j,k}(\tau)\,\dot{\varphi}_j(\tau) + \varphi_k(\tau) + B_{jk}(\tau)\,\varphi_j(\tau) = D_k(\tau). \tag{5}$$

The order of the system (5) is determined by the number of terms of the bilinear series.

In the general case the system (5) is solved through repeated application of the bilinear expansion [3, 4]. If the eigenfunctions of the kernel of Eq. (4) can be used in the first expansion, though, the system (5) breaks down into independent differential equations of the form

$$A_{n,n}(\tau)\,\dot{\varphi}_n(\tau) + \varphi_n(\tau) = D_n(\tau).$$

and the integral for the time functions is written in the general form

$$\varphi_n(\tau) = \left\{\int_0^\tau \frac{D_n(\tau^*)}{A_{n,n}(\tau^*)} \exp\left(\frac{d\tau^*}{A_n(\tau^*)}\right) + \varphi_0\right\} \exp\left(\frac{-\tau}{A_{n,n}(\tau)}\right).$$

In this case the solution of the problem has the form

$$t(x,\tau) = f(x) - \sum_{n=1}^\infty \gamma_n(x)\,\varphi_n(\tau),$$

where the functions $\gamma_n(x)$ are terms of the bilinear series [1].

NOTATION

$t(x, \tau)$, temperature; τ, time; x, coordinate; λ, c, ρ, coefficient of thermal conductivity, specific heat capacity, and density, respectively; $\omega(x, \tau)$, term characterizing the internal heat release; ν = 0, 1, and 2, for problems with plane, axial, and central symmetry.

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